Pointwise Regularity of Fitness Landscapes and the Performance of a Simple ES

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Abstract— **We present a theoretical and experimental analysis of the influence of the pointwise irregularity of the fitness function on the behavior of an (1+1)ES. Previous work on this subject suggests that the performance of an EA strongly depends on the irregularity of the fitness function. Several irregularity measures have been derived for discrete search spaces, in order to numerically characterize this type of difficulty for EA. These characterizations are mainly based on Hölder exponents. Previous studies used however a global characterization of fitness regularity (the global Hölder exponent), with experimental validations being conducted on test functions with uniform regularity. This is extended here in two ways: Results are now stated for continuous search spaces, and pointwise instead of global irregularity is considered. In addition, we present a way to modify the genetic topology to accommodate for variable regularity: The mutation radius, which controls the size of the neighbourhood of a point, is allowed to vary according to the pointwise irregularity of the fitness function. These results are explained through a simple theoretical analysis which gives a relation between the pointwise Hölder exponent and the optimal mutation radius. Several questions connected to on-line measurements and usage of regularity in EAs are raised.**

I. Introduction and Motivation

Intuition, experiments, and theory tends to prove that irregularity is a major cause of convergence pathology for optimisation algorithms in general, and for EAs in particular. Previous work on this topic have established a relation between a measure of the fitness regularity (the global Hölder exponent of the fitness function) and a deception measure [10], [7]. Experimental analyses on Weierstrass functions, have confirmed the theoretical findings.

Weierstrass functions are interesting test functions, as they have a controlled regularity and provide really difficult functions to be tested. Additionnally, the regularity of a Weierstrass function is uniform over its domain. If this is convenient for understanding the behaviour of an EA in controlled environment, this is a limitation in practice: "Real world" fitness functions that one encounters in usual EA applications have variable regularity.

It seems intuitive that the global results obtained previously should apply locally: More precisely, one expects that an EA should more easily locate a maximum lying in a smooth region than a maximum lying in an irregular neighbourhood. This intuition was confirmed trough an experimental analysis, [11]. In that purpose, functions with controlled but variable regularity have been built.

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While it seems clear that local regularity has a major impact on difficulty for EA, it is not however the only factor, and other sources like "epistasy" have to be taken into account [15]. The relationship between irregularity and epistasy has not yet been fully investigated. It seems however probable that these two sources are of different nature ("epistasy is not enough" [14]): some irregular functions are weakly epistatic (for example $f(x_1, ..., x_n) = \sum_1^n e^i x_i$), while some regular functions may be very epistatic (like $f(x_1, ..., x_n) = \prod_1^N x_i$).

Another factor is temporal noise [1], [2]. We do not consider in this paper temporal variations. All functions considered in this paper are fixed, and remain the same during the EA evolution. Regularity variations are considered only with respect to the spatial parameters.

Our work is a contribution to the topic of controlled fitness landscapes, which has been largely developed in the EA community (NK-landscapes and tunable fitness landscapes [14], $(1, \lambda)$-ES on simple function [3]), and on which the behaviour of some simple EA engines are easier to analyse.

Additionally, fitness landscapes involve genetic engine characteristics, that set a specific topology on the definition domain: For a same fitness function, two different EA engines (for example with or without crossover) may have very different behaviour. The terms "fitness landscape" involves both the profile of the fitness function on its definition domain and the search paths produced by the genetic operators. As a consequence, useful quantities for modeling EAs should be measured with respect to this "genetic" topology. For regularity measurements the same holds: Irregularity characteristics must be measured with respect to an underlying measure based on the genetic operators effect. In other terms, the neighbourhood system that serves as a basis for the calculation of Hölder exponents should be linked with transition probabilities via the genetic operators. We should thus talk about fitness landscape irregularity, instead of fitness function irregularity. A first attempt has been done in this direction in [7] for discrete fitness landscapes. The present work deals with continuous functions.

The paper is organized as follows: Section II recalls the basic definitions of Hölder global and pointwise exponents. Section III proposes an analysis that relates the pointwise regularity of a fitness function with the mutation radius of an $(1+1)ES$. Section IV recalls the test-functions built in [11]. The experimental analysis of the proposed adaptive mutation scheme is presented in section V. Conclusions and future work are detailed in section VI.

II. GLOBAL AND POINTWISE REGULARITY

Hölder regularity analysis is an important topic in various fields such as partial differential equations, fractal geometry and signal/image processing ([8]). Hölder regularity allows to quantify in a precise way both the pointwise and global regularity. For our purposes, the following notions will be relevant. To simplify notations, we assume that our signals are nowhere differentiable. Generalization to other signals only requires simple modifications.

Let $\alpha \in (0,1)$, and $\Omega \subset \mathbf{R}$. One says that a function f defined on Ω belongs to $C_l^\alpha(\Omega)$ if:

$$\exists\, C : \forall x, y \in \Omega : \frac{|f(x) - f(y)|}{|x - y|^\alpha} \leq C$$

The supremum of the values α such that f belongs to $C_l^\alpha(\Omega)$ is called the global Hölder exponent of f in Ω. From the definition, it is clear that smaller values of α correspond to more irregular functions.

A pointwise characterization may be obtained as follows: Let $x \in \mathbb{R}$, and s be a real number with $0 < s < 1$. A function $f : \mathbb{R} \to \mathbb{R}$ belongs to $C^s(x)$ if there exist $\delta > 0$ and a constant C_x such that

$$|y - x| \leq \delta \Rightarrow |f(y) - f(x)| \leq C_x |y - x|^s. \qquad (1)$$

The *pointwise Hölder exponent* of f at x, denoted by $\alpha(x)$, is defined to be $\sup\{s : f \in C^s(x)\}$.

Since $\alpha(x)$ is defined at each point, we may associate to f the function $x \mapsto \alpha(x)$ which measures the variation of its regularity with location. Section IV allows one to get an intuitive feeling of Hölder exponents with graphs of functions with prescribed pointwise regularity. Hölder regularity characterization is widely used in fractal analysis because it has direct interpretations both mathematically and in applications. It has been shown for instance that α indeed corresponds to the auditive perception of smoothness for voice signals. Similarly, simply computing the Hölder exponent at each point of an image already gives a good idea of its structure, as for instance its edges [8]. More generally, in many applications, it is desirable to model, synthesize or process signals which are highly irregular, and for which the relevant information lies in the singularities more than in the amplitude. In such cases, the study of the Hölder function is of obvious interest.

III. POINTWISE HÖLDER REGULARITY AND EAS

A. Bounding the "expected fitness progress"

An interesting quantity for the analysis EA behaviour is the expected fitness that can be obtained after the application of genetic operators. This quantity f', called "adjusted fitness" by Goldberg [5], [6], is defined on each point of the search domain. In other words, it is what can be expected as a fitness value from the current point using the genetic operators.

For continuous search space, this quantity is related to the expected progress in one step of an $(1 + 1)ES$ or an $(1 + \lambda)ES$. It is used as a basis for convergence speed analysis on sphere and smooth fitness models [3]. While the calculation of this quantity is difficult for complex fitness functions, the computation of a bound is possible for mutation-only ES on the class of functions described in section II.

Let us consider a *uniform mutation with radius σ*. The mean fitness after mutation is equal to:

$$f'(x) = \frac{1}{2\sigma} \int_{x-\sigma}^{x+\sigma} f(t)dt \qquad (2)$$

The global quantity that was used in [10] as a measure of EA-difficulty is $\Delta f := \max_x(|f'(x) - f(x)|)$. For discrete search spaces and irregular functions, a link with the global Hölder exponent and the parameters of the GA was exhibited.

Since we consider now functions with varying pointwise regularity, it is natural to consider a localized measure of difficulty, *i.e.* $\Delta f(x) := |f'(x) - f(x)|$. Using the pointwise Hölder $\alpha(x)$ exponent of f at x, this quantity may be estimated at any given point x as follows:

$$
\begin{aligned}
\Delta f(x) &\leq \frac{1}{2\sigma} \int_{x-\sigma}^{x+\sigma} |f(t) - f(x)|dt \\
&\leq \frac{C_x}{2\sigma} \int_{x-\sigma}^{x+\sigma} |t - x|^{\alpha(x)}dt
\end{aligned}
$$

$$\text{Thus} \quad \Delta f(x) \leq \frac{C_x \sigma^{\alpha(x)}}{\alpha(x) + 1}$$

This bound suggests that the difficulty varies in a non-linear way with the pointwise regularity of the function. For instance, for a fixed $\sigma < 1$, it decreases when α increases: With small enough mutation radii, smoother functions are easier to handle.

B. A mutation radius varying according to the pointwise regularity

A natural idea is then to choose a location-dependent $\sigma = \sigma(x)$, tuned so as to obtain constant $\Delta f(x)$ along the trajectory. In other words, we require that:

$$\frac{C_x \sigma^{\alpha(x)}}{\alpha(x) + 1} = K$$

where K is a user-defined constant. This leads to the following law of adaptivity of the mutation radius with respect to x:

$$\sigma(x) = \left(\frac{K(\alpha(x) + 1)}{C_x} \right)^{\frac{1}{\alpha(x)}} \qquad (3)$$

Note that the dependency of σ with respect to α is not trivial. In particular, according to the value of the ratio $\frac{K}{C_x}$, the mutation radius may be an increasing (*e.g.* when $\frac{K}{C_x} \leq 0.8$) or decreasing (*e.g.* when $\frac{K}{C_x} \geq 1$) function of the regularity on $[0, 1]$ (the admissible range of α for a non-differentiable function).

In practice, using (3) to tune the value of σ requires the computation of both $\alpha(x)$ and C_x at each point x. This is a

delicate point. A precise estimate would necessitate knowing the value of f at finely sampled points, which is of course not available in applications. We remark however that an initial rough estimation is already sufficient for our purpose. This estimation may be refined as the algorithm proceeds. We thus propose the following procedure: For each point x where the mutation will be applied, we compute the value of the fitness at all points x_i in a small neighbourhood around x. From these values, an estimate of the couple $(C_x, \alpha(x))$ will be obtained as explained below. Equation (3) is then used to compute σ. As the number of generations increases, more points will be investigated, and the estimated $(C_x, \alpha(x))$ will get more precise. In particular, since the algorithm is supposed to visit more often regions of high fitness, the precision will increase preferably exactly at those points we are most interested in: The best estimates will be obtained around maxima of f.

To estimate $(C_x, \alpha(x))$ from the values of f in a neighbourhood of size ε of x, we proceed as follows: Assume one knows $f(x_i)$ for all x_i such that $|x - x_i| \leq \varepsilon$. We compute the oscillations osc_ρ of f defined as:

$$\mathrm{osc}_\rho = \sup_{x_i:|x-x_i|\leq\rho} f(x_i) - \inf_{x_i:|x-x_i|\leq\rho} f(x_i),$$

for $\rho = 1/n, 2/n, \ldots, \varepsilon$, where $1/n$ is the sampling step. The exponent $\alpha(x)$ and the constant C_x are then obtained as the slope and the intercept with the ordinate axis of the linear least square regression of the vector $(\log(\mathrm{osc}_\rho))_\rho$ with respect to $(\log(\rho))_\rho$.

IV. TEST FUNCTIONS WITH CONTROLLED POINTWISE REGULARITY

A. Weierstrass function

In order to precisely and finely investigate the impact of pointwise regularity on the behavior of an EA, we constructed test functions with prescribed Hölder exponent [11]. To make sure that no other factor come into play and thus interfere with the analysis, these functions have been built in the following way.

The basis is a generalized Weierstrass function, which provides a convenient way to control $\alpha(x)$. Let us first recall the definition of the usual Weierstrass function.

$$W_{b,h}(x) = \sum_{i=1}^{\infty} b^{-ih} \sin(b^i x)$$
$$\text{with } b \geq 2 \text{ and } 0 < h < 1$$

The parameter h controls the regularity: The global Hölder exponent of $W_{b,h}$ on, e.g., $[0,1]$, is equal to h. In addition, $\alpha(x) = h$ for all x ([4]). Weierstrass functions are very irregular for small values of h, and become smoother as h tends to 1.

Generalized Weierstrass functions are defined as follows:

$$GW_{b,h}(x) = \sum_{i=1}^{\infty} b^{-ih(x)} \sin(b^i x)$$
$$\text{with } b \geq 2 \text{ and } 0 < h(x) < 1$$

Provided h is differentiable, the pointwise Hölder exponent of $GW_{b,h}$ is $h(x)$ at each x.

Figure 1 displays a generalized Weierstrass function with $h(x) = x$ on $(0, 1)$. One can clearly see the local regularity increasing along the graph. However, an additional feature is present: The local oscillation is large around 0, and decreases as x increases. It is important to note that the variation of the local oscillation is independent from the evolution of $\alpha(x)$. This particular behavior of $GW_{b,h}$ is a nuisance in our case: Since we want to focus on the sensitivity of the EA to pointwise regularity, we need to get rid of other sources of variations, that would perturb our study. We thus deal with a modified version of $GW_{b,h}$ where the local oscillations are normalized. This is explained in details below.
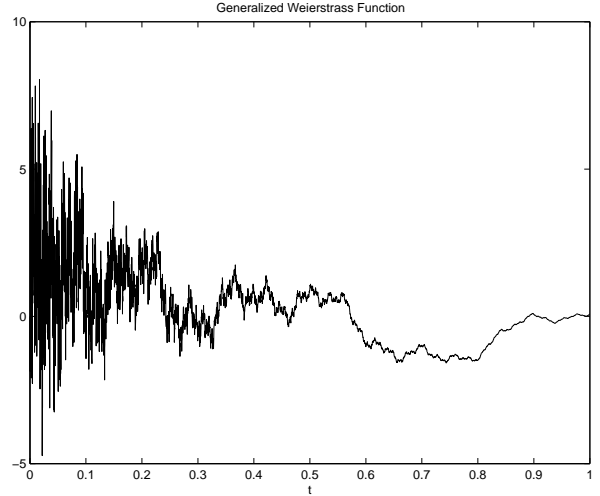

Fig. 1. Generalized Weierstrass function with $h(x) = x$.

B. Test functions

Two test functions have been built with identical features except for the pointwise regularity profile. For obvious reasons, we have constrained the functions to have the same maximum fitness value located at the same point (0, the center of the domain), and a similar underlying smooth (quadratic) component. The irregularity is considered as a "noisy" local perturbation of limited amplitude.

The generalized Weierstrass function is oscillation-normalized as follows. The local mean value and maximal absolute deviation from the mean are computed in a neighbourhood of width ϵ around each point x of the search space $[-0.5, 0.5]$:

$$\mu_\epsilon(x) = \frac{1}{N} \sum_{x_i:|x_i-x|\leq\varepsilon} GW_{b,h}(x_i) \tag{4}$$

$$D_\epsilon(x) = \max_{x_i:|x_i-x|\leq\varepsilon} |GW_{b,h}(x_i) - \mu_\epsilon(x)| \tag{5}$$

where N is the number of points x_i in the ε−neighbourhood of x. The normalised generalized Weiertrass function is then (dotted curves on figures 2 and 3)

$$NW_{b,h}(x) = \frac{GW_{b,h}(x) - \mu_\epsilon(x)}{D_\epsilon(x)}$$

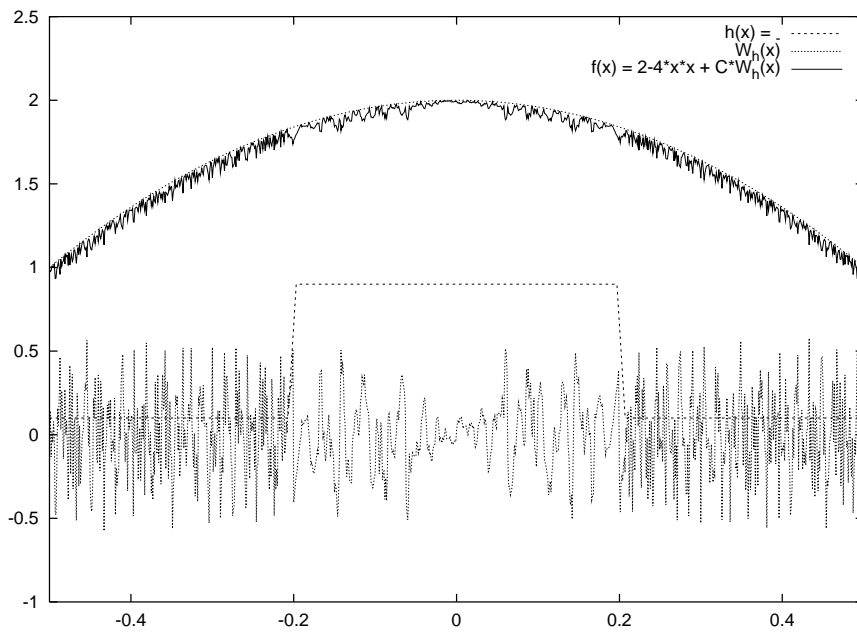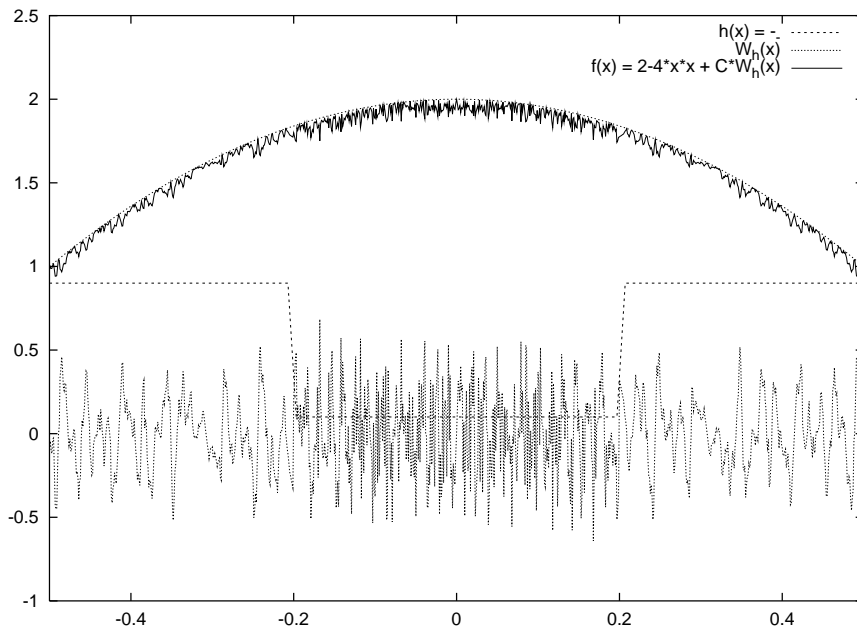Fig. 2.   N(x): The "n" regularity profile function.

Fig. 3.   U(x): The "u" profile regularity function.

The fitness function is finally defined as the smooth trend plus the noisy component with controlled irregularity. It has the following form:

$$f(x) = 2 - 4x^2 - |NW_{b,h}(x)|$$

The noisy component is included as a local perturbation (of small amplitude) that is *subtracted* to the smooth trend, in order to be sure to get the same global maximum at $x = 0$, with the same fitness target value of 2 (since $NW_{b,h}(x)$ always equal 0 at $x = 0$, whatever h). Additionally, each local maximum is located on the smooth trend $2 - 4x^2$.

In the experiments, we consider two profiles:

1) **Favourable case : irregular areas of the function have a low fitness** (Figure2)

$$h(x) = 0.9 \quad \text{if} \quad x \in [-0.2, 0.2]$$
$$h(x) = 0.1 \quad \text{else}$$

2) **Unfavourable case : the most irregular points are located around the global maximum** (Figure3)

$$h(x) = 0.1 \quad \text{if} \quad x \in [-0.2, 0.2]$$
$$h(x) = 0.9 \quad \text{else}$$

Note that both h functions are not differentiable at ± 0.2. At all other points in $[-0.5, 0.5]$, however, h is smooth, and the pointwise Hölder exponent of our fitness function is indeed equal to $h(x)$.

V. EXPERIMENTAL ANALYSIS

The analysis in this section aims at evaluating the efficiency of the adaptive sigma mutation radius of equation (3) on the test functions $U(x)$ and $N(x)$ defined in section IV. In that view, two (1+1)ES have been compared: One with fixed radius mutation (referred to as ES) and one with adaptive radius (referred to as ESadapt). Statistics have been done on 100 runs for each parameter setup.

As a preliminary experiment, we check that $U(x)$ is intrinsically more difficult to optimize than $N(x)$. This is assessed through a pure random search and is illustrated on figure 4. This result comes as a confirmation to our experiments in [11].

Considering the performance of the random search, it has been decided to compare the efficiency of the (1+1)ES at early stages of the search, i.e. with small numbers of evaluations (10 or 20). Longer runs are not interesting, as all algorithms provide fitness results over 1.99.
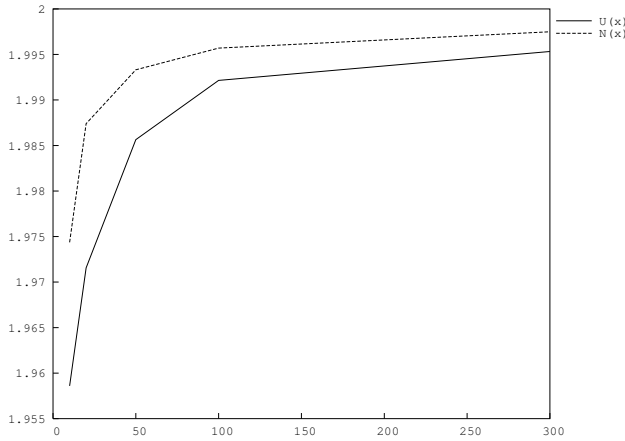
Fig. 4. Mean results (100 runs) of a pure random search on U(x) and N(x): Best fitness (ordinate) *vs* number of trials (abscissa).

A delicate point of the experimentation is the tuning of either the fixed σ (for ES) or the $\alpha(x)$, C_x and K parameters (for ESadapt).

Let us first consider the case of ESadapt. We have fixed $K = 0.1$. This is a reasonable choice in view of the fact that K represents the expected mean fitness variation for a mutation. As explained above, $\alpha(x)$ and C_x should be estimated at each point, using a local sampling. However, our primary aim in this work is to assess the ideal gain entailed by using the adaptive rule (3) for σ. In order to get rid of estimation errors, we have used the known theoretical value $h(x)$ for $\alpha(x)$. As for C_x, it has been experimentally found to be roughly constant for both $U(x)$ and $N(x)$, and approximately equal to 0.15. Figures 5 and 6 give
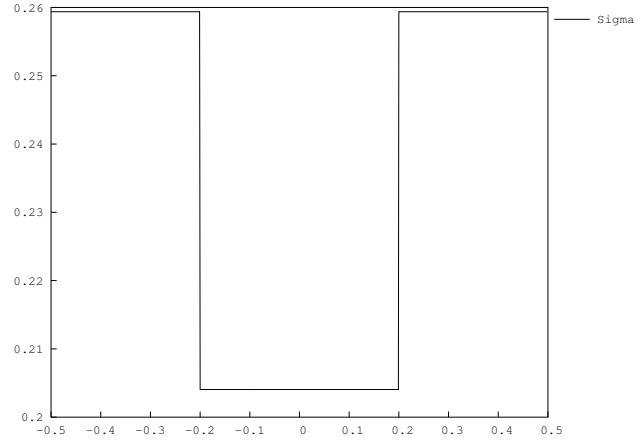
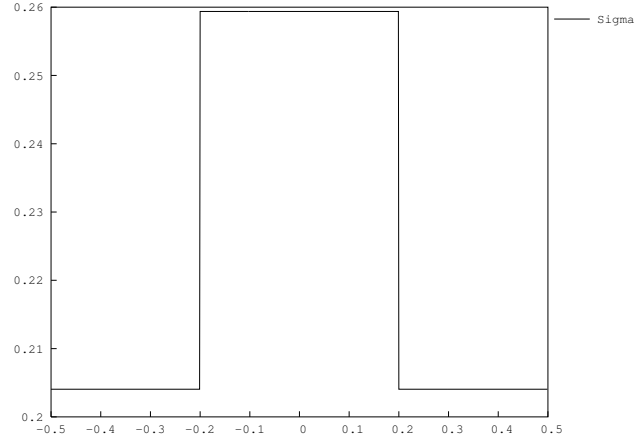Fig. 5. Sigma profile for N(x) over the search space $[-0.5, 0.5]$.

Fig. 6. Sigma profile for U(x) over the search space $[-0.5, 0.5]$.

respectively the mutation radius profiles for the N and U functions based on these parameters settings.

Let us now move to the tuning of σ for ES. Recall that our aim is to compare the efficiency of ES and ESadapt. As there is no obvious way to decide what the optimal σ for ES is, and in order to perform a fair comparison, we chose to let σ vary. More precisely, we have run the experiments on ES for values of σ ranging in a given interval: Curves 7 to 12 show the average best fitness values obtained with ES when σ varies between 0.001 and 0.1. The upper bound 0.1 was chosen in view of the fact that the behaviour of a mutation-only-$(1 + 1)ES$ on the search space $[-0.5, 0.5]$ becomes roughly equivalent to a pure random search algorithm for larger values of σ.

In order to get a meaningful comparison between ES and ESadapt, we define a "mean mutation radius" for ESadapt: This is simply computed as the average of $\sigma(x)$, as given by equation (3), over all x in $[-0.5, 0.5]$. Since this mean mutation radius has no reason to range in the same interval as the fixed σ of ES, we multiply each $\sigma(x)$ by a constant σ_0 so that the mean mutation radius also takes all values in $[0.001, 0.1]$. This rescaling ensures a fair comparison

between the two procedures.

Figures 7 to 12 present the best average fitness obtained after 10, 20 and 50 generation for both algorithms on the functions U and N as a function of σ, *i.e.* the fixed mutation radius for ES, and the mean mutation radius for ESadapt.
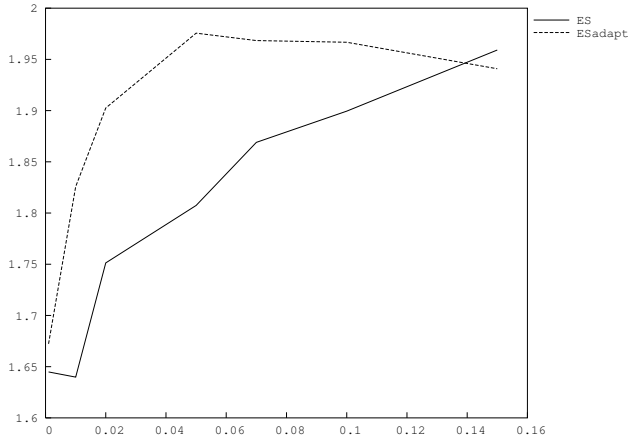

Fig. 7. Result of a 10 generations (1+1)ES on N(x). Mean best fitness (ordinate) *vs* σ (for ES) or mean σ (for ESadapt) (abscissa).
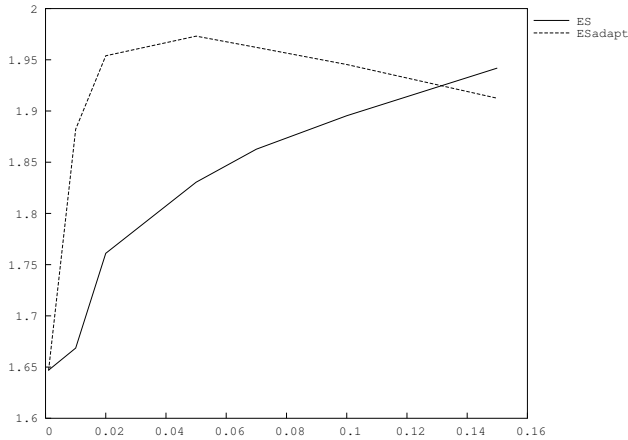

Fig. 8. Result of a 10 generations (1+1)ES on U(x). Mean best fitness (ordinate) *vs* σ (for ES) or mean σ (for ESadapt) (abscissa).

The advantage of ESadapt on ES is particularly clear on the 10 and 20 generations runs. For each optimal tuning of σ, the average best fitness of ESadapt is better that the one of ES. As said above, the difference between the two methods (and also with a pure random search) is less clear for longer runs, due to the small size of the search space. Finally, a striking difference between the behaviours of ES and ESadapt on the favourable $N(x)$ and unfavourable $U(x)$ cases is visible on figures 13 and 14: At the optimal σ (right of figure 13 and near $\sigma = 0.05$ on figure 14), the simple ES has worse performances on the $U(x)$ function, while the performances remain the same for the adaptive strategy.

VI. CONCLUSION AND FUTURE WORK

This work is an extension of the results in [10] to the continuous case. Our results are also coherent with the
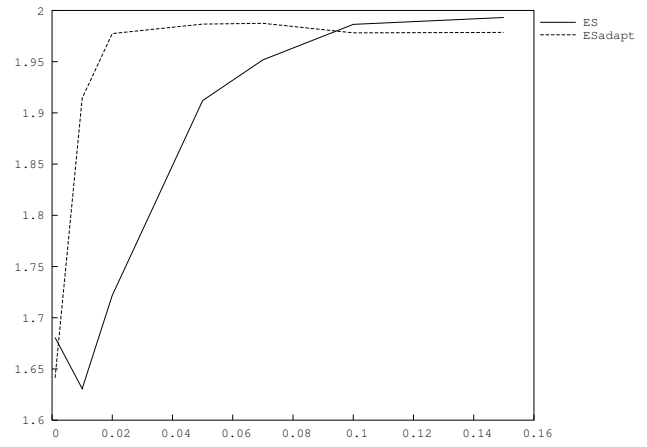

Fig. 9. Result of a 20 generations (1+1)ES on N(x). Mean best fitness (ordinate) *vs* σ (for ES) or mean σ (for ESadapt) (abscissa).
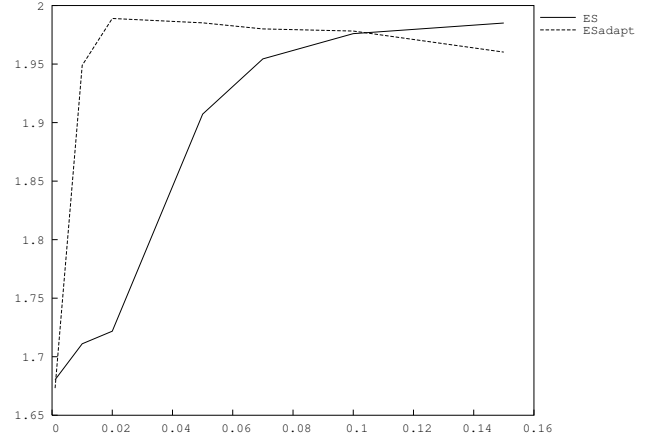

Fig. 10. Result of a 20 generations (1+1)ES on U(x). Mean best fitness (ordinate) *vs* σ (for ES) or mean σ (for ESadapt) (abscissa).

experimental analysis in [11]. In addition, we have proposed a uniform mutation operator with adaptive radius that takes into account the local regularity in an (1+1)ES. Our experiments support the claim that the adaptive scheme is more efficient, and less sensitive to local regularity variations. Future work on this topic will focus on the following aspects:

- From a theoretical viewpoint, we will study the extension of this adaptive scheme to Gaussian mutation (i.e. the classical mutation operator for (1+1)ES). Extension of this analysis to crossover operators seems to be much more difficult to investigate.
- From an applicative viewpoint, an estimation routine for C_x can be easily embedded in a $(1+\lambda)ES$ with almost no loss of computation time. Tests wil be performed in the future. The design of an efficient on-line estimation of the irregularity parameters C_x and $\alpha(x)$ inside a (μ, λ) or $(\mu + \lambda)$ will also be investigated.

This new regularity adaptive scheme should be also compared with other adaptive schemes and auto-adaptive schemes. Each scheme has its proprer balance of calculation cost versus efficiency. An experimental analysis will be
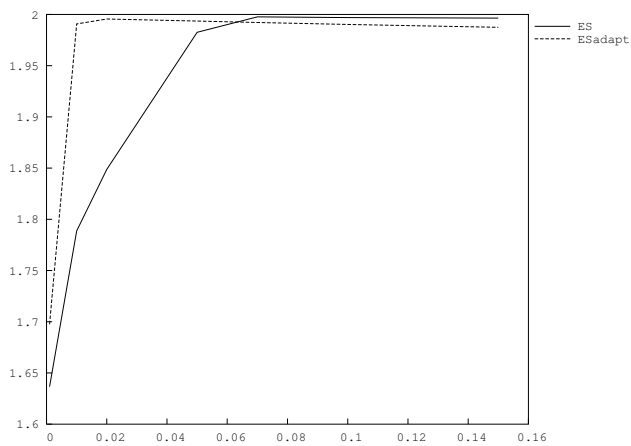
Fig. 11. Result of a 50 generations (1+1)ES on N(x). Mean best fitness (ordinate) *vs* σ (for ES) or mean σ (for ESadapt) (abscissa).
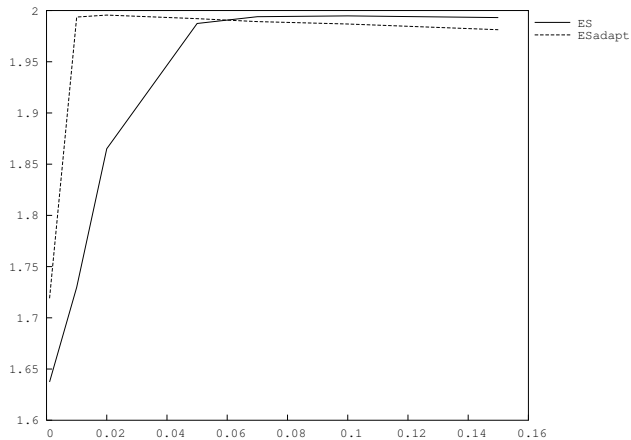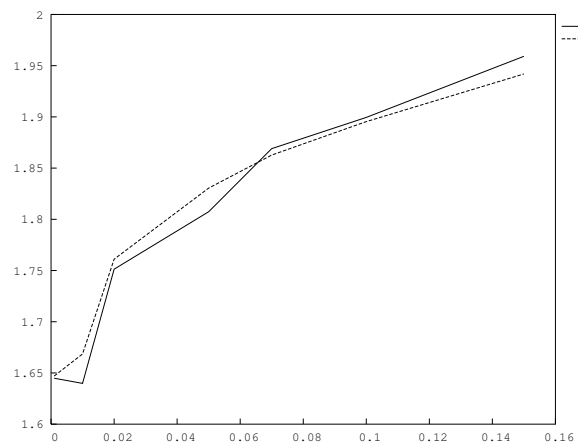

Fig. 13. Comparaison of U(x) and N(x) for the plain (1+1)ES, in 10 generation. Mean best fitness (ordinate) *vs* σ (abscissa).

Fig. 12. Result of a 50 generations (1+1)ES on U(x). Mean best fitness (ordinate) *vs* σ (for ES) or mean σ (for ESadapt) (abscissa).
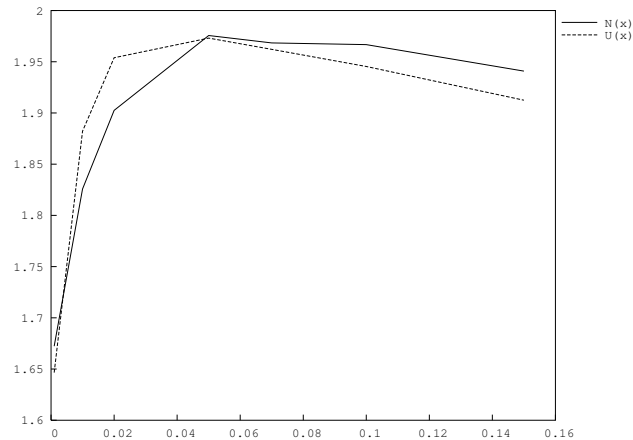

Fig. 14. Comparaison of U(x) and N(x) for the (1+1)ES with adapted mutation radius, in 10 generations. Mean best fitness (ordinate) *vs* mean mutation radius (abscissa).

performed in order to estimate the practical efficiency of our adaptive scheme in the style of [13].

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